LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

UCRL-JRNL-237177

Photon equation of motion with application to the electron's anomalous magnetic moment

A. B. Ritchie

December 10, 2007

Optics Communications

IM-355679
UCRL-

AX-Division
Lawrence Livermore National Laboratory

Burke Ritchie

December 3, 2007

This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.

Lawrence Livermore National Laboratory, P.O. Box 808, Livermore, CA 94551-0808

Photon equation of motion with application to

the electron's anomalous magnetic moment

Burke Ritchie

Lawrence Livermore National Laboratory
Livermore, California 94550

The photon equation of motion previously applied to the Lamb shift is here applied to the anomalous magnetic moment of the electron. Exact agreement is obtained with the QED result of Schwinger. The photon theory treats the radiative correction to the photon in the presence of the electron rather than its inverse as in standard QED. The result is found to be first-order in the photon-electron interaction rather than second-order as in standard QED, introducing an ease of calculation hitherto unavailable.

In a previous paper [1] a photon equation of motion was proposed,

$$\{\nabla^2+\frac{\omega^2}{c^2}+\frac{e}{mc^2}[\vec{\nabla}\cdot\vec{H}+2\vec{H}\cdot\vec{\nabla}+i\vec{\sigma}\cdot(\vec{\nabla}\times\vec{H})+\frac{e}{mc^2}H^2]\}\psi_H=0\ , \qquad (1)$$

where $\vec{\sigma}$ is the Pauli vector and H is a magnetic field. The equation has the Helmholtz form and was used in [1] to infer the Lamb shift for the hydrogen atom. In the present paper we use the equation to calculate the leading anomalous contribution Δg to the electron's magnetic moment,

$$\Delta g=g-2=\frac{\alpha}{2\pi}\ , \qquad (2)$$

which was first calculated by Schwinger [2-3] using QED, where $\alpha=\frac{e^2}{\hbar c}$ is the fine structure constant. This correction to the magnetic moment is related to the divergent electromagnetic contribution to the mass of a free electron, which is also of order α due to cutting off the integral over photon frequency at $\omega_{\max}=\frac{mc^2}{\hbar}$ [4]

such that the product of ω_{max} and the density of photon states is of order c^{-1} or α.

In the present work the anomalous contribution is calculated to first order in H from the quantum average of the interaction Hamiltonian,

$$H' = -\frac{e\hbar}{2mc}\vec{\nabla}\times\vec{A}_r, \qquad (3)$$

where $\vec{A}_r$ is a reaction field operator whose source is a current operator given by the product of the electron's velocity and a total density of states for the electron and photon in the presence of the magnetic field. This ansatz arises from the lesson learned in [1] that the electron and photon share the properties of a single object. The divergences of QED appear to arise from the concept that the electron is a mechanical or matter object interacting with separate light quanta whose frequency spectrum is unrelated to the properties of

the electron.

The third term on the left side of Eq. (1) is zero. The fifth term is zero for the photon in the presence of a uniform magnetic field. The exact solution of Eq. (1) for the remaining terms is,

$$\psi_H = e^{i\vec{\kappa}\cdot\vec{r}} e^{-\frac{eH}{mc^2}|z|} \quad , \qquad (4)$$

for a uniform magnetic field in along z, where $\kappa = \omega/c$. We define a current operator as the Fourier integral of the electron's velocity times the probability $|\psi_H|^2$ of finding the photon at |z|,

$$\vec{j} = \frac{e^{-2\frac{eH}{mc^2}|z|}}{(2\pi)^3} \frac{\hbar}{m} \int d\vec{k}\vec{k} e^{i\vec{k}\cdot\vec{r}} \quad . \qquad (5)$$

Setting $|\psi_H|^2$ equal to unity, the current operator is normalized so that its volume integral is zero,

$$\int d\vec{r}\vec{j} = \frac{\hbar}{m} \int d\vec{k}\vec{k}\delta(\vec{k}) = 0 \ , \qquad (6)$$

as appropriate for an electron at rest. We are justified

in dropping H-dependent terms in the normalization of the current and in the operations which follow below except that of the cross product in H’ [Eq. (3)], which contributes to first order in H, the order in which the magnetic moment of the electron is measured. Notice that the current vanishes when the photon is found at large longitudinal distances from the electron. Thus the magnetic-moment anomaly is a photonic effect but according to the present theory not in the sense of the emission and absorption of virtual photons.

Knowing the current from Eq. (5) the reaction field in Eq. (3) is calculated by solving Maxwell’s equation. Then the quantum average of the interaction Hamiltonian H’ is calculated from the Fourier transform of H’, a result made possible by having expressed the current in terms of its Fourier representation in Eq. (5).

Finally the anomalous contribution to the magnetic moment is found from the sum of this result over physically accessible states of the electron.

The transverse components of the reaction field, which are the only components to contribute to first order in H, are,

$$A_{r,x,y} = \frac{1}{(2\pi)^3}\frac{e\hbar}{mc}\int d\vec{k}(k_x,k_y)\int d\vec{r}'\frac{e^{i\vec{k}\cdot\vec{r}'}e^{-2\frac{eH}{mc^2}|z'|}}{|\vec{r}-\vec{r}'|} \cong \frac{1}{2\pi^2}\frac{e\hbar}{mc}\int d\vec{k}\,\frac{(k_x,k_y)}{k^2}e^{i\vec{k}\cdot\vec{r}}e^{-2\frac{eH}{mc^2}|z|} \quad . \qquad (7)$$

The k-space integral can be evaluated exactly,

$$\frac{1}{2\pi^2}\int d\vec{k}\,\frac{(k_x,k_y)}{k^2}e^{i\vec{k}\cdot\vec{r}} = \frac{i(\cos\phi,\sin\phi)}{\pi}\int_{-\infty}^{\infty}dk_z\int_0^{\infty}dk_\rho\frac{k_\rho^2}{k_\rho^2+k_z^2}J_1(k_\rho\rho)e^{ik_z z} = i(\cos\phi,\sin\phi)\int_0^{\infty}dk_\rho k_\rho J_1(k_\rho\rho)e^{-k_\rho|z|} =$$

$$i(\cos\phi,\sin\phi)\frac{\rho}{(\rho^2+z^2)^{\frac{3}{2}}} \quad . \qquad (8)$$

Notice that the cross product on the right side of Eq. (3) contributes terms to first order in H arising from the z-derivative of the $|\psi_H|^2$ term on the right side of Eq. (7). The Fourier transform of the reaction field

is,

$$\tilde{H}_{r,x,y} = \int d\vec{r}e^{-i\vec{k}\cdot\vec{r}}(\nabla\times\vec{A}_r)_{x,y} = \frac{4\pi e\hbar eH}{m^2c^3}(\sin\phi_k,-\cos\phi_k)\int_{-\infty}^{\infty}dz\int_0^{\infty}d\rho\frac{\rho^2}{(\rho^2+z^2)^{\frac{3}{2}}}J_1(k_\rho\rho)e^{-ik_z z}e^{-2\frac{eH}{mc^2}|z|} \cong$$

$$\frac{8\pi e\hbar eHk_z}{m^2c^3}(\sin\phi_k,-\cos\phi_k)\int_0^\infty d\rho\,\rho K_1(k_z\rho)J_1(k_\rho\rho)=\frac{8\pi e\hbar eH}{m^2c^3}(\sin\phi_k,-\cos\phi_k)\frac{k_\rho}{k_\rho^2+k_z^2}\,, \tag{9}$$

where again we have kept only terms first order in H.

The scalar product of Eq. (9) with Pauli's vector is written,

$$\vec{\sigma}\cdot\vec{H}_r=\begin{pmatrix}0 & \tilde{H}_{r,x}-i\tilde{H}_{r,y}\\ \tilde{H}_{r,x}+i\tilde{H}_{r,y} & 0\end{pmatrix}=\frac{8\pi e\hbar eH}{m^2c^3}\frac{k_\rho}{k_\rho^2+k_z^2}\begin{pmatrix}0 & ie^{-i\phi_k}\\ -ie^{i\phi_k} & 0\end{pmatrix} \tag{10}$$

The eigenvalues of the matrix in Eq. (10) are $\varepsilon=\pm 1$; thus the diagonal interaction is independent of the azimuthal angle ϕ_k. The anomalous contribution to the magnetic moment is given directly by the upper-sign diagonal of $\vec{\sigma}\cdot\vec{H}_r$ summed over states of the electron and divided by H,

$$\Delta g=\frac{8\pi e^2\hbar}{(2\pi)^3m^2c^3}\int_{-\infty}^{\infty}dk_z\int_0^{\frac{mc}{\hbar}}dk_\rho\frac{k_\rho^2}{k_\rho^2+k_z^2}=\frac{e^2\hbar}{\pi m^2c^3}\int_0^{\frac{mc}{\hbar}}dk_\rho k_\rho=\frac{\alpha}{2\pi}\ , \tag{11}$$

where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant. Notice that the integral over the radial electron wave number is cut off at $\frac{mc}{\hbar}$ analogously to cutting off the photon frequency in mass renormalization at $\frac{mc^2}{\hbar}$. The physical inference is that the radial electron momentum and photon momentum are equivalent or $\hbar k_\rho = \hbar\omega / c = mc$. Notice finally that Δg is differential and isotropic in the azimuthal angle ϕ_k.

In conclusion we have used the photonic theory presented in [1] to calculate the leading anomalous contribution to the electron’s magnetic moment. As in [1] the calculation requires that we know the electronic current and the equation of motion of the photon in

the presence of the electron. These requirements lead to a Lamb shift and anomalous magnetic moment which are calculated to first order in the electron-photon interaction rather than to second order as in QED, thereby introducing an ease of calculation hitherto unavailable.

Acknowledgements. This work was performed under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. DE-AC52-07NA27344

References

[1] B. Ritchie, Optics Communications **280**, 126 (2007). The change of sign of the magnetic field in Eq. (6b) of the referenced paper has no physical consequences for the Lamb shift application since the physically appropriate sign of the two imaginary diagonal contributions to the interaction is that chosen for the absorption of the photon by the bound electron.

[2] J. Schwinger, Phys. Rev. **73**, 416 (1948); J. Schwinger, Phys. Rev. **82**, 664 (1951).

[3] P. J. Mohr and B. N. Taylor, Revs. Mod. Phys. **77**, 1 (2005).

[4] H. A. Bethe, Phys. Rev. **72**, 339 (1948); W. H. Louisell, **Quantum Statistical Properties of Radiation** (Wiley, New York, 1973), pp. 285-296.